The first part of Lec 9 is in the Lec 8 notes.

HSW converse

$M - X^n - Q^n - \hat{M}$

$$(1-\epsilon) nR - 1 \underset{\substack{\text{classical} \\ \text{data processing}}}{\le} I(M;\hat{M}) \le I(X^n;\hat{M}) \underset{\substack{\text{quantum} \\ \text{data processing}}}{\le} I(X^n;Q^n) \underset{\substack{\text{same} \\ \text{as classical}}}{\le} \sum_{i=1}^{n} I(X_i;Q_i) \le n\,C(\mathcal{N})$$

q channels

$$\chi(\mathcal{N}\otimes\mathcal{M}) \ge \chi(\mathcal{N}) + \chi(\mathcal{M})$$

sometimes $=$ "additive"

1) $\mathcal{N}$ is entanglement-breaking. QCQ
$\mathcal{N}(\rho) = \sum_k \mathrm{tr}[\rho M_k]\,\sigma_k$

2) $\mathcal{N}$ is depolarizing $\mathcal{N}(\rho) = (1-p)\rho + p\frac{I}{d}$

3) erasure $\mathcal{N}(\rho) = (1-p)\rho + p|e\rangle\langle e|$

4) unital qubit channel $\mathcal{N}(I) = I$

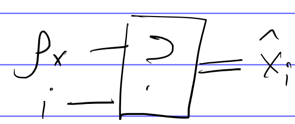
5) pure loss bosonic channel

However
$\exists\ \mathcal{N}$ s.t. $\chi(\mathcal{N}^{\otimes 2}) > 2\chi(\mathcal{N})$

Applications of HSW converse

1) random access coding

$x \in \{0,1\}^m \rightarrow \rho_x$ in n qubits



$m=2$, $n=1$ can achieve $\Pr[\hat{x}_i = x_i] = \cos^2(\pi/8) \approx 0.85\ldots$
classically can't beat $1/2$

However quantumly error $\epsilon \Rightarrow n \ge m(1-H_2(\epsilon))$

Lemma given σ_0, σ_1, $M_0 + M_1 = I$ s.t. $\mathrm{tr}\, M_b\sigma_b \ge 1-\epsilon$
$\sigma = \frac{\sigma_0+\sigma_1}{2} \Rightarrow S(\sigma) \ge \frac{S(\sigma_0)+S(\sigma_1)}{2} + 1 - H_2(\epsilon)$

Pf $\quad \rho_{XQ} = \dfrac{|0\rangle\langle 0| \otimes \sigma_0 + |1\rangle\langle 1| \otimes \sigma_1}{2}$

$$I(X;Q) = S(Q) - S(Q|X)$$
$$= S(\sigma) - \frac{S(\sigma_0) + S(\sigma_1)}{2}$$

$$\geq I(X;b) \qquad b = \text{measurement outcome}$$
$$= H(X) - H(X|b)$$
$$\geq 1 - H_2(\epsilon)$$

RAC

$$\rho = \frac{1}{2^m} \sum_{x \in \{0,1\}^m} |x\rangle\langle x| \otimes \rho_x$$

$$S(Q \mid X_1 \ldots X_k) \geq S(Q \mid X_1 \ldots X_{k+1}) + 1 - H_2(\epsilon)$$

$$\Rightarrow \quad m(1 - H_2(\epsilon)) \leq S(Q) \leq n$$

1.5) Quantum State learning

given state ρ on n qubits

need $\exp(n)$ bits to describe

What about most measurements $M \sim \mathcal{D}$?

(Consider 2-outcome measurements $\{M, I-M\}$ for simplicity)

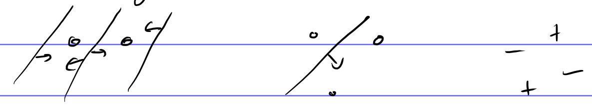
Think of ρ as a map from $\mathcal{M} \to [0,1]$

for simplicity we will only care if $\operatorname{tr} M\rho > 1-\epsilon$ or $< \epsilon$

VC dimension (S= states) = measure of complexity

= largest set that is "shattered" by S

e.g.



given measurements $M_1, \ldots, M_d \in \mathcal{M}$ and outcomes $o_1, \ldots, o_d \in \{0,1\}$

$\exists ? \ \rho$ s.t. $|\operatorname{tr} \rho M_i - o_i| \leq \epsilon$

by RAC bound, this is possible only if $d = O(n)$

$\Rightarrow$ ρ can be learned with $O(n)$ samples

next time we'll see that $\Omega(2^n)$ or $\Omega(4^n)$

samples are needed to output $\hat{\rho}$ s.t. $\|\rho - \hat{\rho}\|_1 \leq \epsilon$