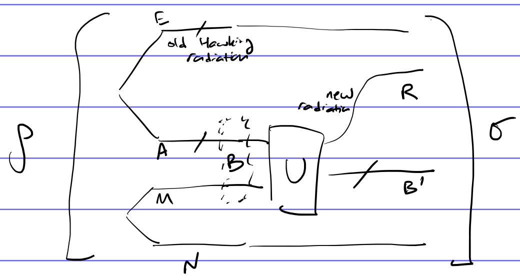
Application Black holes as mirrors Hayden-Preskill 0708.4025



$$\mathbb{E}\,\|\sigma_{NB'} - \sigma_N \otimes \sigma_{B'}\|_1^2 \le \frac{d_N d_{B'}}{d_R}\, \overbrace{\operatorname{tr}\rho_{BN}^2}^{\nearrow \frac{d_N}{d_B}} \approx \frac{d_N^2}{d_R^2}$$

$$d_B = d_{B'} d_R$$

$$d_N = O(1) \to \text{need } d_R = O(1)$$

Analogy to QECC

$$|\psi\rangle = |\Phi\rangle_{RA_1}^{\otimes a}\ |\Phi\rangle_{A_2B}^{\otimes b}$$

merging needs a qubits from $A \to B$ or b from $B \to R$

If we apply $U_A \sim$ Haar then <u>any</u> $a+\delta$ or $b+\delta$ qubits work

Application of channel coding

$$\mathcal{N} : A' \to B \qquad V_{\mathcal{N}} : A' \to BE$$

$$I_c(\phi_{A'}, \mathcal{N}) = S(B)_{V(\phi)} - S(E)_{V(\phi)} \qquad \text{"coherent information"}$$

$$|\gamma\rangle_{ABE} = (I_A \otimes V_{A' \to BE})\,|\phi\rangle_{AA'}$$

$$|\Gamma\rangle_{SB^nE^n} = \left(\Pi^n_{A,\delta} \otimes I_{B^nE^n}\right) |\gamma\rangle^{\otimes n} = \left(I_S \otimes V^{\otimes n}\right) |\Phi\rangle_{SS'}$$

fix projector $P_{S \to R}$ sample $U_S \sim$ Haar

$$|\psi\rangle_{RB^nE^n} = \sqrt{\frac{d_S}{d_R}}\left(PU \otimes I_{B^nE^n}\right)|\Gamma\rangle = \left(I_R \otimes V^{\otimes n} U^T\right)|\Phi\rangle_{RR'}$$

$$\mathbb{E}\,\|\psi_{RE^n} - \psi_R \otimes \psi_{E^n}\|_1^2 \le d_R \operatorname{rank}(\psi_{E^n}) \operatorname{tr}\psi_{SE^n}^2$$

$$\le d_R \exp\left(-n\left(S(B)_\phi - S(E)_\phi - 2\delta\right)\right)$$

works if $\log d_R < n(I_c - 2\delta)$

$\Rightarrow$ avg-case coding. Can be strengthened to worst case.

Coherent information

can be negative

$$Q(\mathcal{N}) = \lim_{n\to\infty} \frac{1}{n} \max_{\rho} I_c(\rho, \mathcal{N}^{\otimes n})$$

$\max_{\rho}$ makes it non negative.

Example $\quad D_\eta(\rho) = (1-\eta)\rho + \frac{\eta}{3}(X\rho X^\dagger + Y\rho Y^\dagger + Z\rho Z^\dagger)$

$$I_c(I/2, D_\eta) = 1 - H_2(\eta) - \eta \log 3$$

entangled input can help

$\eta \approx 0.19 \quad I_c(I/2, D_\eta) < \frac{1}{5} I_c\left(\frac{|00000\rangle\langle 00000| + |11111\rangle\langle 11111|}{2}, D_\eta^{\otimes 5}\right)$ "degenerate codes"

Superactivation : $\exists\, \mathcal{N}_1, \mathcal{N}_2$ with $Q(\mathcal{N}_1) = Q(\mathcal{N}_2) = 0$ $\quad Q(\mathcal{N}_1 \otimes \mathcal{N}_2) > 0$

reasons for 0 capacity

- PPT $\quad \mathcal{N}$ is PPT if $(\mathcal{N} \otimes I)(\Phi)$ is PPT
- antidegradability
  $\mathcal{N} = \operatorname{tr}_E V_\mathcal{N}$ $\quad \exists\, \mathcal{M} : E \to B$ s.t. $\mathcal{N} = \mathcal{M} \circ \mathcal{N}^c$
  $\mathcal{N}^c = \operatorname{tr}_B V_\mathcal{N}$
  (Doesn't survive back communication.)

R

B