Monogamy without symmetry

$$I(A;B_1 \ldots B_k) = \sum_{i=1}^{k} I(A;B_i \mid B_{<i}) \qquad B_{<i} = B_1 \ldots B_{i-1}$$

$$\leq 2 \log d_A$$

$$\Rightarrow \mathop{\mathbb{E}}_i I(A;B_i|B_{<i}) \leq \frac{2\log d_A}{k}$$

$$\boxed{\begin{array}{l}\text{"squashed entanglement"} \\ E_{sq}(\rho_{AB}) = \min \frac{1}{2} I(A;B|E)_{\tilde\rho} \\ \qquad \tilde\rho_{ABE} : \tilde\rho_{AB} = \rho_{AB}\end{array}}$$

so what if $I(A;C|B) \leq \epsilon$?

what if $\underset{\text{CMI}}{\underset{\shortparallel}{I(A;C|B)}} = 0$?

classically $\text{CMI} = 0 \Leftrightarrow p(x,y,z) = e^{-E_1(x,y) - E_2(y,z)}$ etc...

quantumly

$$I(A;C|B) = 0 \Leftrightarrow \rho = \exp(\log \rho^{AB} + \log \rho^{BC} - \log \rho^{B})$$

$$\Leftrightarrow B \cong \bigoplus_\alpha B_\alpha^L \otimes B_\alpha^R \qquad \rho = \bigoplus_\alpha p_\alpha \, \sigma_\alpha^{AB_\alpha^L} \otimes \omega_\alpha^{B_\alpha^R C}$$

^ classical variable

$$\Leftrightarrow \exists R_{B\to BC} \text{ s.t. } \rho_{ABC} = (\mathrm{id}_A \otimes R_{B\to BC})(\rho_{AB})$$

called "quantum Markov state"

$$\Rightarrow \rho_{AC} \in \text{Sep}$$

Approx case?

$$\text{CMI}(\rho) \neq \min_{\sigma \text{ Markov}} D(\rho \| \sigma) \qquad \text{unlike classical case}$$

but $\text{CMI} \geq -2 \log \max_R F(\rho_{ABC}, R_{B\to BC}(\rho_{AB}))$

i.e. $\exists R_{B\to BC}$ s.t. $F(\rho_{ABC}, R(\rho_{AB}))^2 \geq \exp(-\text{CMI})$

[Fawzi-Renner '14]

More generally

$$D(\rho\|\sigma) - D(N(\rho) \| N(\sigma)) \geq -2 \log \max_{R \text{ s.t. } R(N(\sigma)) = \sigma} F(\rho, R(N(\rho)))$$

Pf sketch

q state redistribution needs $n\, I(A:C|B)$ qubits (and some ebits) for $AC\text{-}B \Rightarrow A\text{-}CB$

replace communication with max. mixed states. Use dF reduction to get single-letter protocol with rel. ent. error $I(A:C|B)$

Many-body physics and q. information

H acts on $(\mathbb{C}^d)^{\otimes n}$

$H = \sum_{S \subset [n]} h_S$ h_S acts on qudits in S, eg $H = \sum_{i \sim j} h_{ij}$ or $\sum_{ijk} h_{ijk}$ etc..

Bosons $d \to \infty$ Fermions later

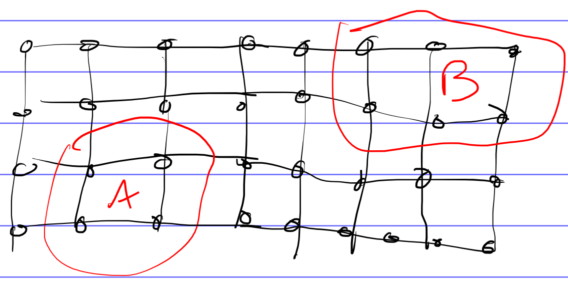
objects of study: e^{-iHt}, $e^{-H/T}$, $|\psi_{gs}\rangle$

complexity? BQP $\approx$ QMA

but these are worst case – often things are easier

big question How does locality of H affect locality of e^{-iHt}, $e^{-\frac{H}{T}}$ & $|\psi_{gs}\rangle$?

one answer: two-point correlations



operators X_A, Y_B
$A = \text{supp } X$ etc.

$X_A(t) = e^{iHt} X_A e^{-iHt}$

correlations $\langle X_A Y_B \rangle - \langle X_A \rangle \langle Y_B \rangle$

another answer: full description of the state $\cong$ general observables

e.g. consider the toric code