$$H = \sum_{S \subset V} h_S$$

How does locality of H affect locality of e^{-iHt}, etc..

<u>Lieb – Robinson bound</u>

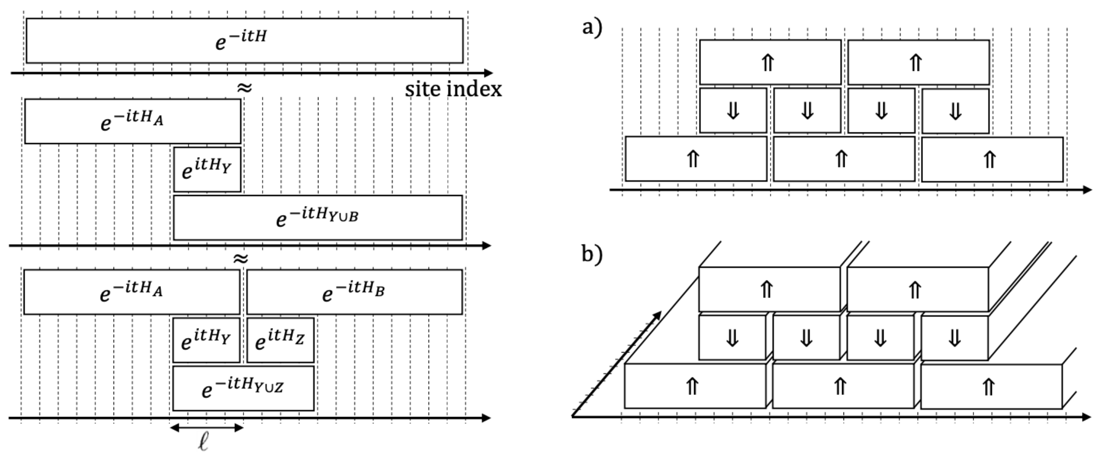
$\mu > 0 \qquad \alpha := \max_i \sum_{S \ni i} \|h_S\| \cdot |S| \, e^{\mu \, \mathrm{diam}(S)}$

A_X, B_Y supported on $X, Y \subset V$

$\| [A_X(t), B_Y] \| \le 2 \|A_X\| \|B_Y\| |X| \exp(-\mu \, \mathrm{dist}(X,Y)) (e^{2\alpha|t|} - 1)$

$\exp\left(-\mu\left(\mathrm{dist} - \underbrace{\frac{2\alpha}{\mu}}_{V_{LR}} t\right)\right) \Rightarrow$ approx light-cone with speed V_{LR}

<u>Application</u> Simulating lattice Hamiltonians [Haah, Hastings, Kothari, Low '18]



$$\| U_t^{A\cup B} (U_t^B)^\dagger U_t^{B \cup C} - U_t^{A \cup B \cup C} \| \le O(e^{-\mu \, \mathrm{dist}(A,C)}) \sum_{S \text{ on bdy}(AB:C)} \|h_S\|$$

$$W_t := e^{it(H_{AB} + H_C)} e^{-itH_{ABC}} = (U_t^{AB+C})^\dagger U_t^{ABC}$$

$$\partial_t W_t = U_t^{AB+C} \underbrace{(H_{ABC} - H_{AB} - H_C)}_{H_{bd}} (U^{AB+C})^\dagger W_t$$

$$\approx U_t^{B+C} H_{bd} (U_t^{B+C})^\dagger W_t$$

$\exp(-\mu \, \mathrm{dist}(A,C))$

Application correlation decay in gapped ground states

$|\psi_0\rangle = |\psi_{gs}\rangle$

Suppose H has gap g, i.e. energies $E_0 < E_1 = E_0 + g \le E_2 \le \dots$

then $|\langle A_x B_y\rangle - \langle A_x\rangle\langle B_y\rangle| \lesssim e^{-\frac{lg}{2v_{LR}}} \|A\| \, \|B\|$

example TFIM $\quad H = -J\sum_i Z_i Z_{i+1} - B\sum_i X_i$

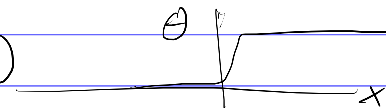
$B \gg 0$ unique g.s. $|+\rangle^{\otimes N} = |\rightarrow\rightarrow\rightarrow\rightarrow \cdots \rightarrow\rangle$ no correlations

$|B| \ll J$ $\quad \dfrac{|0\rangle^{\otimes N} \pm |1\rangle^{\otimes N}}{\sqrt{2}}$ have exp. small gap long-range correlations $\langle Z_i Z_j\rangle = 1$

Pf of correlation decay

Assume WLOG that $\langle A\rangle = \langle B\rangle = 0$ $\quad E_0 = 0$

$\langle E_i | B^+ | E_j\rangle := \langle E_i | B | E_j\rangle \, \theta(E_i - E_j)$



$B^+ |\psi_0\rangle = B|\psi_0\rangle \qquad \langle\psi_0| B^+ = 0$

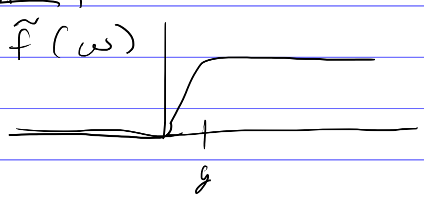
$\langle AB\rangle = \langle AB^+\rangle = \langle [A, B^+]\rangle$

but B^+ is nonlocal

$$\int f(t)\, B(t)\, dt = \sum_{i,j} |E_i\rangle\langle E_j| \, B_{ij} \int dt\, f(t)\, e^{it(E_i - E_j)}$$

$$= \sum_{i,j} |E_i\rangle\langle E_j| \, B_{ij}\, \tilde{f}(E_i - E_j)$$

goals for f



$\tilde{f}(\omega) \approx 0 \qquad \omega < 0$

$\tilde{f}(\omega) \approx 1 \qquad \omega > g/2$

say erf

$|f(t)|$ small if $t \gtrsim 1/g$

then $B^+ \approx \int dt\, f(t)\, B(t)$

$[A, B^+] \approx \int dt\, f(t)\, [A, B(t)]$

small if $t \gtrsim 1/g$ $\qquad \lesssim e^{-\mu l}$ if $t \le 1/g$

balancing terms $\Rightarrow$ error $\exp\left(\dfrac{-lg}{2V_{LR}}\right)$