Local description of a state

$|\psi_1\rangle \otimes \cdots \otimes |\psi_n\rangle$ good but this is demanding a lot

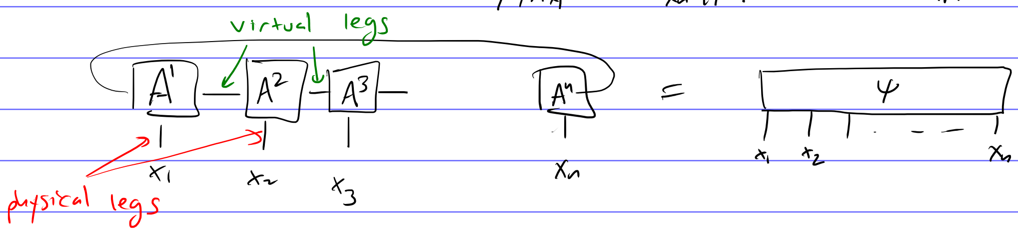
$\sum_{i=1}^{k} |\psi_{1,i}\rangle \otimes \cdots \otimes |\psi_{n,k}\rangle$ ok but often $k \sim \exp(n)$, e.g. $|\Phi\rangle_{12} \otimes |\Phi\rangle_{34} \otimes \cdots$

Matrix Product States

$|\psi\rangle = \sum_{x \in \{0,1\}^n} |x_1, \dots, x_n\rangle \, \mathrm{tr}[A^1_{x_1} A^2_{x_2} \cdots A^n_{x_n}]$ or $\langle \phi | A^1_{x_1} \cdots A^n_{x_n} | \phi \rangle$

dim $A^i_x \le r$ = "bond dim"

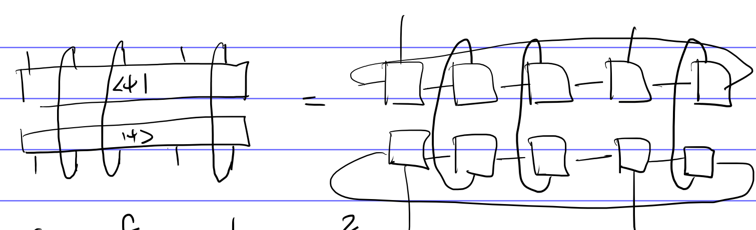
nr^2 storage



"tensor network"

MPS description enables calculation of local observables

e.g. $\mathrm{tr}_{[n]-\{1,4\}} \psi$ =



requires multiplying matrices of dim r^2

In general, effort for most tasks is poly(n,r)

MPS have SR (Schmidt rank) $\le r$ for cuts of the form $1..k : k+1, \dots, n$

conversely any state with $\max_k$ rank $\psi_{1..k} \le r$ is an MPS with bond dim r.

e.g. $|\Phi\rangle_{12} \, |\Phi\rangle_{34} \cdots$

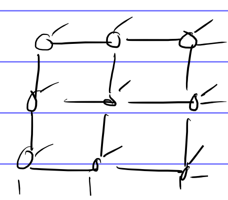
$\frac{|0\rangle^{\otimes n} + |1\rangle^{\otimes n}}{\sqrt{2}}$

$|W\rangle = \frac{|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle}{2}$

all have $r = 2$

dim > 2?

tensor networks



- imply SR bounds
- low space representation

but generally lack efficient algorithms

$|\psi\rangle \approx$ MPS $\iff$ $S_0(\psi_{1\ldots k}) \le \log r$

or $S_0^{\epsilon}(\psi_{1\ldots k}) = \min\{ S_0(\rho) : \|\rho - \psi_{1\ldots k}\|_1 \le \epsilon \}$

raises question of <u>area laws</u>

$|\psi\rangle \in (\mathbb{C}^d)^{\otimes V}$ meaning a qudit on each $v \in V$

$S(\psi_A) \le c\,|\partial A|$ "area law" $S(\psi_A) \sim c|A|$ "volume law"

means entanglement is short range

$\approx$ stronger than correlation decay.

e.g. $|\psi\rangle \sim$ Haar has volume law ent and low correlations

"data hiding"

What we know about area laws:

1D $S(\psi_A) \lesssim \dfrac{\log^3 d}{g}$

2D frustration-free locally gapped $S(\psi_A) \le |\partial A|^{1+\delta}$ if $g \sim O(1)$

<u>Area laws from AGSP</u>

$K|E_0\rangle = |E_0\rangle$

$\Pi = 1 - |E_0\rangle\langle E_0|$ $K\Pi \le \Delta\,\Pi$

$SR(K) \le D$

<u>Claim</u> $D\Delta \le 1/2 \Rightarrow \exists$ prod state $|\phi\rangle$

$|\langle\phi|E_0\rangle|^2 \ge \dfrac{1}{2D}$

$$|\langle\phi|E_0\rangle|^2 \ge \mu \Rightarrow S(|E_0\rangle) \lesssim \frac{\log \mu^{-1}}{\log \Delta^{-1}} \log D$$

<u>exact area law?</u>

$S(\psi_A) = c|\partial A| - \gamma + o(1)$

γ from TEE (topological entanglement entropy) (i.e. terms $\to 0$ as $|\partial A| \to \infty$)